Green roofs to mitigate the Urban Heat Island

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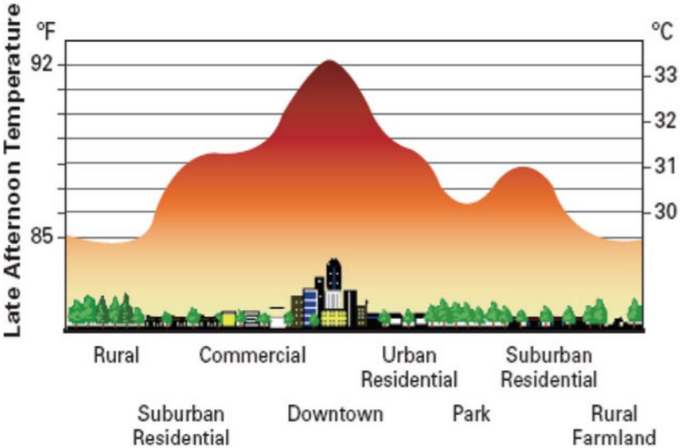
Outline

What is the Urban Heat Island?

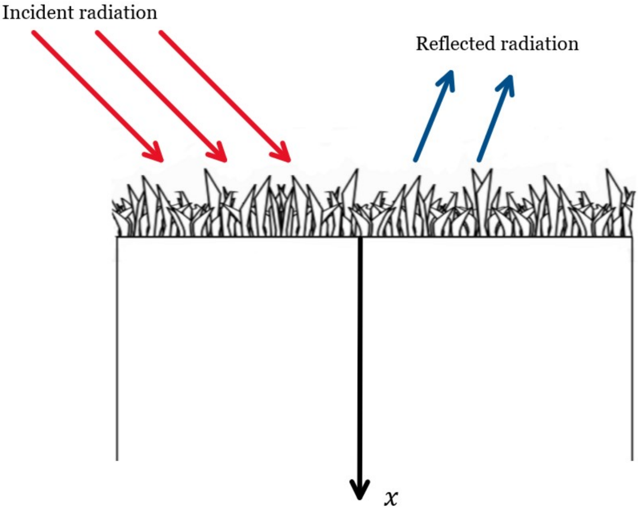
The **Urban Heat Island** is a phenomenon where the temperature of urban areas and a region of a city (generally the inner city) is higher than the surrounding less dense outskirts. This phenomenon is mostly observed in the late afternoons.



What are green roofs?

A **green roof** or living roof is a roof of a building that is partially or completely covered with vegetation.

Break-up of the problem



Break-up of the problem

We consider the heat equation for the temperature of the soil of a green roof at depth x and time t:

$$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \tag{1}$$

subject to three boundary conditions:

- The temperature at time $t = 0$ at any point x is equal to T_{avg}: $T(0, x) = T_{avg}$
- The temperature at position $x \to \infty$ will be equal to T_{avg} : $T(t, x \to \infty) = T_{avg}$
- The temperature at position $x = 0$ is dependent on the heat flux into the soil at $x = 0$

Break-up of the problem

In order to describe our last boundary condition, we will need to calculate the net heat flux on the green roof.

The components of the heat flux we will consider:

- Solar heat flux from shortwave radiation: $Q_s - \gamma Q_s$
  - Q_s = Irradiance recieved by surface
  - γ = albedo, $0 < \gamma < 1$
- Heat flux due to black body radiation: $\epsilon\sigma(T^4 - T_a^4)$
  - σ = Stefan-Boltzmann constant
  - ϵ = emmisivity
  - T = soil temperature
  - T_a = ambient air temperature
- Heat flux by evapotranspiration: $L_v \dot{d} \rho_w$
  - L_v = latent heat of vaporisation of water
  - $\dot{d}$ = evapotranspiration rate
  - ρ_w = water density
- Heat flux transfer by convection:
  - h = convective heat transfer coefficient

Simplifying some terms

We simplify the contribution to the net flux from the black body radiation by expanding the term as a series around T_a:

$$f(T) = f(T_a) + (T - T_a)\frac{\partial f}{\partial T} + (T - T_a)^2 \frac{\partial^2 f}{\partial T^2} + \dots$$

Since the terms of $T - T_a$ with orders higher than 1 is sufficiently small, it can be discarded. Furthermore, $f(T_a) = 0$. This gives:

$$f(T) \approx (T - T_a)\frac{\partial f}{\partial T} = (T - T_a)(4T_a^3)$$

Simplifying some terms

Combining all the contributions to the heat flux, including the simplified contribution from the black body radiation, we find the net heat flux:

$$Q_{net} = Q_s - \gamma Q_s - \epsilon\sigma(T - T_a)(4T_a^3) - h(T - T_a) - L_v \dot{d}\rho_w \tag{2}$$

We also have the general equation for heat flux into a surface:

$$Q_{net} = -k\frac{\partial T}{\partial x}(t, 0) \tag{3}$$

Problem Statement

Solve:

$$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \tag{4}$$

given:

$$T(0, x) = T_{avg} \tag{5}$$

$$T(t, x \to \infty) = T_{avg} \tag{6}$$

$$-k \frac{\partial T}{\partial x}(t, 0) = Q_s - \gamma Q_s - \epsilon\sigma(T - T_a)(4T_a^3) - h(T - T_a) - L_v \dot{d} \rho_w \tag{7}$$

Non-dimensionalization

In order to simpify our problem we apply the following non-dimensionalization:

$$\bar{x} = \frac{x}{L}$$

$$\bar{t} = \frac{t}{\tau}$$

$$\bar{T} = \frac{T - T_{avg}}{\Delta T}$$

ΔT, L and τ the scales to be determined.

Non-dimensionalization

After the non-dimensionalization, our problem statement now reads:
Solve:

$$\frac{\partial \bar{T}}{\partial \bar{t}} = \frac{\partial^2 \bar{T}}{\partial \bar{x}^2} \tag{8}$$

given:

$$\bar{T}(0, \bar{x}) = 0 \tag{9}$$

$$\bar{T}(\bar{t}, \bar{x} \to \infty) = 0 \tag{10}$$

$$insert - equation \tag{11}$$